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Assignment - 1

Ques 1:- Let A be a set with n elements. Justify that the power set $P(A)$ has 2^n elements.

Soln:- For $0 \le k \le n$, then there are ${}^nC_k$ ways to select subsets having k elements of the set A.

So, total no. of subsets of

$$A = {}^nC_0 + {}^nC_1 + {}^nC_2 + \text{-----} + {}^nC_{n-1} + {}^nC_n = 2^n$$

Now, expanding binomial expansion we gets,

$$(x+y)^n = {}^nC_0 x^n y^0 + {}^nC_1 x^{n-1} y^1 + \text{-----} + {}^nC_{n-1} x^1 y^{n-1} + {}^nC_n x^0 y^n$$

$$x = 1, \; y = 1$$

$$\therefore 2^n = {}^nC_0 + {}^nC_1 + \text{-----} + {}^nC_{n-1} + {}^nC_n$$

Hence
Proved

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Ques2:- Let A, B, C be three non-empty sets, justify the following -

(i) $A-(B\cap C) = A-B \cup (A-C)$

(ii) $(A^c \cup B^c)^c \cup (A^c \cup B)^c = A$

(iii) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(iv) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol^n:- (i) LHS :- $A-(B\cap C)$

$= A \cap (B \cap C)^c$

$= A \cap (B^c \cup C^c)$

$= \cancel{(A \cap A) \cap (B^c}$

$= (A \cap B^c) \cup (A \cap C^c)$

$= (A-B) \cup (A-C)$

$=$ R.H.S.

R.H.S. :- $(A-B) \cup (A-C)$

$= (A \cap B^c) \cup (A \cap C^c)$

$= A \cap (B^c \cup C^c)$

$= A \cap (B \cap C)^c$

$= A-(B \cap C)$

$=$ L.H.S

(ii)

(iii)

sets,

L.H.S

(ii) $(A^c \cup B^c)^c \cup (A^c \cup B)^c$

$= (A^c)^c \cap (B^c)^c \cup (A^c)^c \cap B^c$

(De-morgan's law & involution law)

$= (A \cap B) \cup (A \cap B^c)$

$= A \cap (B \cup B^c)$ (Distributive law)

$= A \cap U$ (complement law)

$= A$ (Identity law)

(iii) Let $(x,y) \in A \times (B \cup C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in B$ or $x \in A$ and $y \in C$

$\Rightarrow (x,y) \in A \times B$ or $(x,y) \in (A \times C)$

$\Rightarrow (x,y) \in (A \times B) \cup (A \times C)$

$\Rightarrow A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$ — (1)

Conversely-

Let $(x,y) \in (A \times B) \cup (A \times C)$

$\Rightarrow (x,y) \in (A \times B)$ or $(x,y) \in (A \times C)$

$\Rightarrow x \in A$ and $y \in B$ or $x \in A$ and $y \in C$

$\Rightarrow x \in A$ and $(y \in B$ or $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cup C)$

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$\Rightarrow (x,y) \in (A \times (B \cup C))$

$\Rightarrow (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$ —— ②

from ① and ② -

$$A \times (B \cup C) = (A \times B) \cup (A \times C)$$

(iv) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Let $(x,y) \in A \times (B \cap C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow x \in A$ and $x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow (x,y) \in (A \times B)$ and $(x,y) \in (A \times C)$

$\Rightarrow (x,y) \in (A \times B) \cap (A \times C)$

$A \times (B \cap C) \subseteq (A \times B) \cap (A \times C)$ —— ①

Conversely -

Let $(x,y) \in (A \times B) \cap (A \times C)$

$\Rightarrow (x,y) \in (A \times B)$ and $(x,y) \in (A \times C)$

$\Rightarrow (x \in A$ and $y \in B)$ and $(x \in A$ and $y \in C)$

$\Rightarrow x \in A$ and $(y \in B$ and $y \in C)$

$\Rightarrow x \in A$ and $y \in (B \cap C)$

$\Rightarrow (x,y) \in (A \times (B \cap C))$

$(A \times B) \cap (A \times C) \subseteq A \times (B \cap C)$ —— ②

Ques 3:- ldow

rela

A

Solⁿ:- For

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Ques 4:- Let

the

refl

(a) R_1

Solⁿ:- → (

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from ① & ② -

$$A \times (B \cap C) = (A \times B) \cap (A \times C)$$

②

Ques 3:- How many reflexive & symmetric relations there are on a set A with n elements?

Soln:- For reflexive relation $= 2^{n^2 - n}$

$= 2^{n(n-1)}$

For symmetric relation $= 2^n \times 2^{\frac{n(n-1)}{2}}$

Ques 4:- Let $A = \{1, 2, 3, 4\}$. Identify which of the following relations on A are reflexive, symmetric, transitive and/or anti symmetric.

(a) $R_1 = \{(1,1), (1,2), (2,1), (2,3), (3,3), (4,1), (5,2)\}$

Soln:- → $(2,2), (4,4) \notin R_1$

∴ so R_1 is not reflexive

→ $(2,3) \in R_1$ but $(3,2) \notin R_1$

∴ so R_1 is not symmetric

→ $(1,2) \in R_1$ and $(2,3) \in R_1$ but $(1,3) \notin R_1$

so R_1 is not transitive

→ $(1,1)$ and $(3,3) \in R_1$

so R_1 is anti symmetric

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(ii) $R_2 = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$

→ $(1,1), (2,2), (3,3), (4,4) \in R_2$

so R_2 is reflexive

→ $\forall (a,b) \in R_2 \Rightarrow (b,a) \in R_2$

so R_2 is symmetric

→ $\forall a, b, c \in R_2$

$aR_2b, bR_2c \Rightarrow (a,c) \in R_2$

so R_2 is transitive

→

(iii) $R_3 = \{(2,1), (3,1), (3,2), (4,1), (4,2), (4,3)\}$

→ $\forall a \in A$

$(a,a) \notin R_3$

so R_3 is not reflexive

→ $\forall a, b \in A$

$\Rightarrow aR_3b$ but $(b,a) \notin R_3$

so R_3 is not symmetric

→ $\forall a, b, c \in A$

aR_3b and $bR_3c \Rightarrow (a,c) \in R_3$

Ques 5:-

Soln:-

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So R_3 is transitive.

Ques 5:- Let $A = \mathbb{Z}$, the set of integers, for $a, b \in A$, define

$a \sim b \Leftrightarrow 6$ divides $b - a$

Justify that $\sim$ is an equivalance relation. Also, write all elements of the set $\mathbb{Z}_6$.

Soln:- Since 6 divides $a - a = 0$, for any $a \in \mathbb{Z}$, it follows that $a \sim a$ so that $\sim$ is a reflexive relation.

Suppose, $a \sim b$

So we have, $a - b = 6k$ (k = some int.)

But then,

we also have, $b - a = 6(-k) \Rightarrow b \sim a$

So, $\sim$ is a symmetric relation.

To prove transitivity of $\sim$,

let $a \sim b$ and $b \sim c$

$a - b = 6k_1$ and $b - c = 6k_2$

$k_1, k_2 \in \mathbb{Z}$ (Int.

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Adding both -

$a - c = (a-b) + (b-c) = 6(k_1 + k_2)$

which shows that $a \sim c$. therefore, $\sim$ is an equivalance relation

Finally, recall that the set $\mathbb{Z}_6$ consists of associated equivalance classes. That is,

we have -

$\mathbb{Z}_6 = \{[0], [1], \ldots [5]\}$

where

$[k] = k + 6\mathbb{Z}$; for $k = 0, 1, \ldots 5$

Ques 6:- Let $A = \{1, 2, 3, 4\}$. Use Warshall's algo to complete the transitive closure of reln R on A given by

$R = \{(1,1), (1,3), (2,1), (2,2), (3,3), (3,4)\}$

Soln:- $W_0 = M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

Step 1 - Compute W_1

Location of non-zero entries in C_1 :- 1, 2

$R_1 = 1, 3$

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Make entries '1' at position (1,1) (1,3) (2,1) (2,3)

$$W_1 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 2 :- Compute W_2

Loc. of non-zero entries in C_2 :- 2

R_2 :- 1, 2, 3

Make entries '1' at pos. (2,1) (2,2) (2,3)

$$W_2 = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Step 3 :- Compute W_3

Loc. of non-zero entries in C_3 :- 1, 2, 3

R_3 :- 3, 4

Make entries '1' at pos (1,3) (1,4) (2,3) (2,4) (3,3) (3,4)

$$W_3 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

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Step 4 :- Compute W_4

Loc of non-zero entries in C_4 :- 1, 2, 3

R_4 :- –

No entries change

$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$R^* = \{(1,1), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,3), (3,4)\}$

Ques 7 :- In all the posets considered below the partial order $\leq$ is the relation "divides".

(i) Draw the Hasse diagram of the posets $(P_1, \leq)$ and $(P_2, \leq)$, where

$P_1 = \{1, 2, 3, 4, 6, 7, 8, 9, 12, 18, 24\}$

$P_2 = \{3, 4, 12, 24, 48, 72\}$

(ii) Find the maximal and minimal elements of the poset $(P, \leq)$; where

$P = \{2, 4, 5, 10, 12, 20, 25\}$

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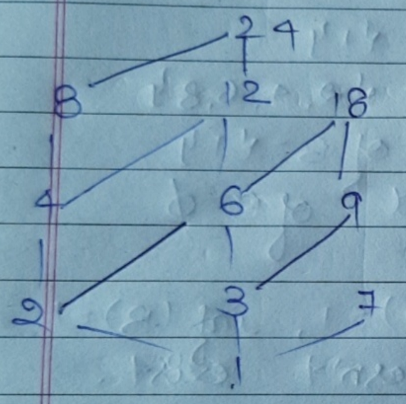
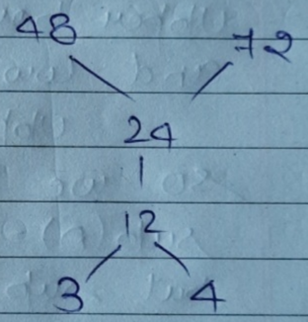
(iii) Let $P = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{1, 2\}$ and $B = \{3, 4, 5\}$. Find the lower and upper bounds of the subsets A and B of the Poset $(P, \leq)$. Also find the $\sup(A)$, $\inf(A)$, $\sup(B)$ and $\inf(B)$.

Soln:-





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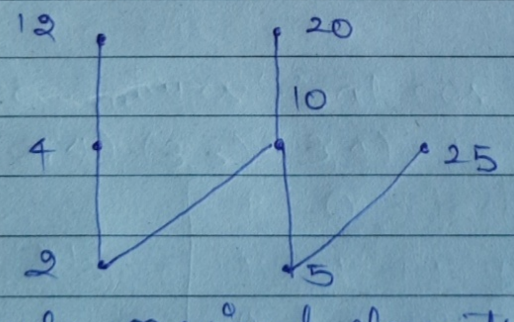

Clearly maximal elements are 12, 20, 25 and minimal elements are 2, 5

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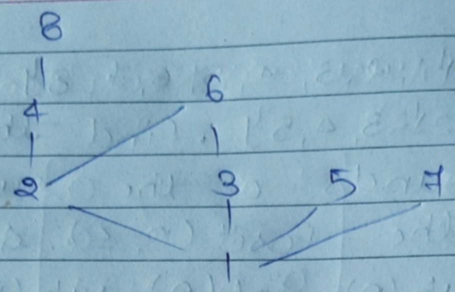
(iii)



The lower and upper bound for the subsets A = {1, 2} & B = {3, 4, 5} of P are :-

Lower bound of A = {1}

Upper bound of A = {2, 4, 6, 8}

and lower bound of B = {1}

upper bound of B = ϕ

So we have

sup(A) = 2 , inf.(A) = 1, inf (B) = 1

but sup(B) is doesn't exist.